Dynamic Bus Routing Problem for Evacuation

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Abstract

Evacuation is an important preventative decision before disaster occurs. Evacuation process has heavily dependent on automobile. While, in recent years, public transit attracts more and more attention but has been extensively used in evacuation planning. This paper proposed a one-stage multiclass evacuation model based on Cell Transmission Model(CTM) where evacuation buses is assigned to dangerous area to evacuate people. The model dynamically captures the interaction between normal vehicles and buses units where the priority of buses and First-In-First-Out process is observed. A heuristic method incorporating bender decomposition is developed to achieve a approximating optimal problem solutions. The computational results from the real case study justify the interaction we observed before.

Keywords: evacuation planning, bus routing, mixed-integer programming

1 Introduction

Disasters threaten human life in many different ways, whether they are wild fires, earthquakes, tornados, volcanic eruptions, tsunamis or nuclear leaks. Both natural and man-made disaster are hard to predict and damaging to affected societies. When disaster occurs, one of the critical procedures is to evacuate people from affected areas. Especially for the disasters that can be predicted a few hours before, people are informed and scheduled to evacuate before it strikes. Among significant natural disasters, hurricanes have long been regarded as predictable since they occur seasonally. Evacuation plans for coastal cities threatened by hurricanes have become more and more important over time, since a bad decision in the evacuation could lead to irredeemable damage. In 2005, Hurricane Katrina struck New Orleans. The government failed to efficiently evacuate people from the city. More than 1245 people died in the hurricane and subsequent flood[17]. One month after Hurricane Katrina, another hurricane called Rita landed in Louisiana. It is reported that 113 people died and 4526 single-family dwellings were destroyed by Rita[1]. A decade after the hurricane, experts started to investigate evacuation plans that the government used in hurricane Katrina and they proved that loss of life could be reduced if a proper evacuation plan is deployed. test

When Katrina landed, there were 10,000 citizens who could not access reliable personal transport. Most people evacuated by their own vehicle. Meanwhile, there were 500 public transit and school buses could be used to evacuate people, but the government had no plan for transit evacuation. If the government fully used public transit to evacuate people, many lives could have be saved[20].

Numerous evacuation planning studies focus on auto-mobile-based evacuation. The research on transit-based evacuation is relatively less. Public transit outperforms automobiles in their mass capacity and in greater compliance to manager. Moreover, special needs people (eg. physical disability, older adults, residents) who do not have reliable modes of transportation, can be evacuated by buses. By deploying public transit in evacuation planning, road congestion will be significantly alleviated since less automobiles are loaded into the network.

Some work had been done on transit evacuation, most of which assumes static network and does not consider the interaction between buses and automobile vehicles. In this paper, we develop a one-stage Mixed Integer Linear Programming(MILP) model with multiclass vehicles on a dynamic network, where evacuation buses and normal vehicles evacuate simultaneously. A hurricane scenario is considered and citizens are asked to evacuate out of dangerous areas as much as possible before the hurricane strikes while bus size, loading and unloading process is strictly captured.

This paper advanced the modelling of evacuation planning by making following specific contributions.

- A multiclass evacuation model is developed in which the interaction between automobile and evacuation transit is captured.
- Several behaviors of buses are tracked: Buses units are counted in the traffic flow and follow first-in-first-out(FIFO) procedure; a bus's loading and unloading time is captured depending on how many people it carries. Each bus is given priority based on the people it carries.
- In stead of using bi-level structure which is general used in evacuation scenario, a dynamic single stage evacuation model is developed based on cell transmission model(CTM).

The remainder of this study is organized as follows. In Section 2, we present a literature review on public transit-based evacuation model and the Cell Transmission Model. In Section 3, a novel single stage mixed-integer dynamic bus evacuation model has been proposed and the details of model has been discussed. In Section 4, we model the evacuation plan on a small network and the network behavior has been observed. In Section 5, an advanced decomposition algorithm has been used to solve the problem. In Section 6, details of the application of the proposed model to a real world case study have been described. Section 7 mentions some comments about the CTM and proposed the future work.

2 Literature Review

Evacuation can be categorized into notice evacuation(terrorist attack, nuclear leak) and no-notice evacuation(earthquake, hurricane, tsunami). No-notice evacuations requires an immediate evacuation plan for a mass evacuation[9]. In contrast, notice evacuation planning has more time for preparation, because citizens are informed a few hours before the disaster strikes. A smart evacuation plan can save many lives. Therefore, using available supply to minimize loss within limited time becomes a crucial problem for a manager.

Numerous studies have examined strategies for evacuation planning, which can be classified into supply-based and demand-based. Supply-based strategies aim to facilitate an increase in the road throughput and to reduce traffic congestion during evacuation. For example, contraflow is employed during an evacuation to change the direction of traffic flow and temporally increase the road capacity(Tudyes and Ziliaskopoulos[26]; Xie et al.[29]). Similarly, cross elimination is used to avoid traffic conflicts at the intersections([10];Zhao[32]). In terms of demand-based strategy, staging and routing structure is commonly applied in the evacuation model. Evacuees are loaded in the network intermittently , so that the road congestion is reduced.(Bish et al.[6] Tudyes and Ziliaskopoulos[27] Bish and Sherali[5];Chen and Zhan[8]).

Another line of evacuation planning research is determining the travel plan for the evacuation vehicle. In that case, the problem is determining optimal destinations, travel routes, departure time for evacuation vehicles. The evacuation vehicle consider in this paper is public transit which is already been proved to be efficient from many research. Public transit, compared with personal vehicles, carry more people and are advanced in four aspects including mitigation, preparedness, response, and recovery[25]. Among the public transit evacuation studies, simulation approach is used for its capability of capturing behavior characteristics of drivers and traffic flow dynamics. Kaisar et al.[15] use microscopic simulation model to analysis the real-life traffic flow and find the best stop location for public transit. Kimms[16] formulate an optimization-based simulation model to schedule traffic route during evacuation. Although high fidelity detail can be captured by simulation approach, the simulation method is better to used for evaluating and assessing rather than evacuation planning[28].

On the other hand, optimization based approach generate evacuation plan for public transit by searching optimal solution to the model. Such model looking for the best evacuation plan for buses so that minimize the total evacuation time, minimize the network clearance time or maximize the number of people evacuated. Kaisar[15] developed an optimization model to find the optimal stops for evacuation buses, so that overall benefit is maximized. Kulshrestha[18] proposed a robust optimization model to determine the pick-up location for buses to minimize total evacuation time. In terms of determing the optimal route for buses, which is the focus of this paper, the Vehicle Routing Problem(VRP) and variant VRP is commonly used. Abdelgawad and Abdulhai[2] developed an Optimal Spatio-Temporal Evacuation model based on the Multiple Depots, Time Constraints and Multiple pick-up and Delivery VRP. Bish[4] developed a bus evacuation problem(BEP), which is similar to the split delivery multi-depot VRP with inter-depot routes(SDMDVRPI). Sayyady and Ekisoglu[25] modeled the bus routing by assigning the traffic flow on available buses while traffic flow is obtained by simulation model. Chen et al.[7] developed a model that determines the optimal evacuation routes and number of evacuees to pick up for public transit on VRP with time windows(VRPTW) model.

The VRP based evacuation model is a traditional way to formulate the static evacuation model. However, static model assumes constant travel time, which is unrealistic during evacuation process. Additionally, some evacuation resources are movable or need to be dynamically allocated. In such a case, dynamic modelling for evacuation is indispensable.

The Cell Transmission Model(CTM) is popular for dynamic traffic modelling. CTM is introduced by Daganzo[11][12] and implemented into linear programming by Ziliaskopoulos[33]. The basic idea is to discrete the network into small pieces of homogeneous cells with time spanning into small time steps. Traffic flow between cells and the number of vehicles in cells follows hydrodynamic traffic flow and density relationships, as proposed by Lighthill and Whitham[19], Richards[24](usually called LWR model). Since the CTM precisely captures the traffic flow backward propagation and traffic flow spillover and can model an actual network with reasonably fidelity, numerous dynamic evacuation models are developed based on CTM.

Generally , CTM is used as lower level modeling in evacuation planning to determine the traffic flow assignment while the upper level determines the evacuation strategy(Chiu et al.[9];Yazici and Kaan[31];Yao et al.[30]). One stage formulation for evacuation related decision making is sparse. The only paper in literature review that model evacaution with one stage CTM is He[13]. This paper formulates a MILP based on CTM to allocate resources during evacuation. However, model CTM based model in one stage makes problem extraordinary diffcult to solve, especially when it comes to real case network. To reduce the complexity of the MILP model, He[14] developed an earliest arrival flow model later, and this model can be solved efficiently when using bender decomposition. The model in this paper is somewhat similar to the resource allocation model in He's paper. But in our model, the allocation units are buses, which evolved more complicated issues: destination for each trip(for pick-up and loading point), which routes to choose and how many people are loaded into each bus.

In the many transit-based evacuation studies, the evacuation of automobile vehicles and evacuation buses are considered as two separate problems[3]. As these are related in real evacuation planning. In this paper, the interaction between buses and normal vehicles is captured and modeled during evacuation. The buses are responsible for transferring people from the endangered zone and the evacuees either evacuate by bus or by themselves. By scheduling the evacuation plan for both buses and evacuees, the total travel time is minimized. Several decision need to be made for buses: to which destination buses should go, how many people a bus should pick up, and which route a bus should take.

To solve the these questions, a one-stage multiclass evcaution model is developed. Two kinds of vehilce are assigned based on CTM where evacuation buses and normal vehicles evacuate simultaneously. Bus capacity and size are considered, the loading and unloading process is strictly captured(loading time monotone increasing with the number of people), buses are dynamically routed accoding to traffic and demand. The queuing process is captured as the only interaction between buses and normal vehicles. In other words, buses follows the First-In-First-Out(FIFO) rules and cannot drive through the traffic flow. In this paper, citizens are assumed to fully comply with the evacuation plan so that the evacuation objective can be achieved.

3 Formulation

In this section, a MILP model is developed for dynamic bus-based evacuation planning. Mesa-Arango and Ukkisuri[23] have investigated traffic assignment problem incorporating car-truck interaction based on CTM. Liu[21] model the interaction in terms of moving bottleneck caused by slow-moving buses. Both of them consider the movement of buses as traffic flow and model the interaction between bus flow and normal vehicle flow. In this paper, interaction between buses and traffic flow is formulated as queuing process. This paper develops a multiclass evacuation model in which evacuation buses follow FIFO rules when travelling through the network and obtain different priority based on different number of people carried. Some evacuees are evacuated by buses and the rest of them evacuate by themselves using automobiles. The model assumes that the evacuees follow the instruction completely to achieve system optimal.

3.1 Notation

The proposed Dynamic Bus Routing problem is based on Cell Transmission Model. The network is divided into homogeneous cells C, such that the length of cell is equal to the distance a vehicle travel with free-way speed in one time step. The Cells are connected by connectors ε which represent the traffic flow travel between cells. Cells are categorized into three types by different performance: the subset of source cells, $C_R \subset C$, the subset of intermediate cells $C_I \subset C$ and the subset of sink cells $C_S \subset C$. Source cells represent dangerous area that generate demand; Sink cells represent the shelters; Intermediate cells consist the body of network. Let $\Gamma^+(i)$ denote the set of successor cells of cell $i \in C$ and $\Gamma^-(i)$ denote the set of predecessor cells of cell $i \in C$. Traffic vehicles moves with direction from predecessor cells to successor cells.

3.2 Decision variable

Each cell has two kinds of variable that dynamically describe state of system. x_i^t represents the number of vehicle in each cell i at time t. y_{ij}^t represents number of vehicles move from cell i to cell j at time t. The bus assignment variable is b_{pi}^t, which equal to 1 if bus p is in cell i at time t. Two auxiliary variables E_{pi}^t and L_{pi}^t are used to denote entering and leaving status for bus in each time step.

C = all cells in the network
C_R = subset of source cells
C_I = subset of intermediate cells
C_S = subset of sink cells
$\Gamma^(n)$ = set of adjacent cells of cell $n \in C$
$\Gamma^+(i)$ = set of successor cells of cell $i \in C$
$\Gamma^-(i)$ = set of predecessor cells of cell $i \in C$

Parameters
N_i = the capacity of a cell $i \in C$
q_i^t = the maximum inflow or outflow of cell $i \in C$ at time t
δ = the ratio $\delta = w/v$, where w is propagation backward speed and v is free-flow speed

d_i^t = the demand in source cell i at time t
P = the number of available bus
C_p = the capacity of bus p
τ = maximum time limits a bus can stay in each single cell
ψ_p = relative occupancy ratio of bus p.

Decision Variables
x_i^t = indicates number of vehicles in cell $i \in C$ at time $t \in T$
y_{ij}^t = indicates number of vehicles travel from cell i to cell j at time t
b_{pi}^t = indicates whether or not bus p is in cell i at time t
E_{pi}^t = indicates whether or not bus p is entering cell i at time t
L_{pi}^t = indicates whether or not bus p is leaving cell i at time t

3.3 Objective function

The objective of the proposed model is to minimize the total system travel time of all evacuees in the evacuation process. The evacuee includes the people evacuate by normal vehicles and people evacuate by bus.

$$\sum_{t \in T} \left(\sum_{i \in \mathcal{C} \setminus \mathcal{C}_{\mathcal{S}}} x_i^t + \ell \sum_{0}^{t} \sum_{p \in P} \left(\sum_{i \in C_R} b_{pi}^t - \sum_{i \in C_S} b_{pi}^t \right) \right) \Delta t \tag{1}$$

To simplify the problem, the number of evacuees is divided by the pre-assumed average number of people in normal vehicles n, so that the total evacuation time for people evacuated by auto-vehicle can be represented as $\sum_{i \in \mathcal{C} \setminus \mathcal{C}_{\mathcal{S}}} x_i^t \Delta t$. In the same way, the the number of people evacuate by buses is derived by dividing the same number n. The number of people in bus p at time t can be derived from difference value between the accumulated loading time and accumulated unloading time before t.

3.4 Constraints

The body of constraints can be divided into three parts. The first part is CTM constraints. The second part is loading and unloading process constraints. The third part is Bus Routing Problem constraints.

Flow Conservation Constraints
Cell Transmission Model contains two part of constraints. Flow conservation constraints and flow restriction constraints. To cooperate buses activity into

CTM, we reformulate flow conservation constraints as follow:

$$x_i^t = x_i^{t-1} - \sum_{j\in\Gamma^+(i)} y_{ij}^{t-1} + \sum_{k\in\Gamma^-(i)} y_{kj}^{t-1} + d_i^{t-1} - \sum_{p\in P} b_{pi}^{t-1} \quad \forall i \in C_R, t \in T \tag{2}$$

$$x_i^t = x_i^{t-1} - \sum_{j\in\Gamma^+(i)} y_{ij}^{t-1} + \sum_{k\in\Gamma^-(i)} y_{kj}^{t-1} \quad \forall i \in C_I, t \in T \tag{3}$$

$$x_i^t = x_i^{t-1} - \sum_{j\in\Gamma^+(i)} y_{ij}^{t-1} + \sum_{k\in\Gamma^-(i)} y_{kj}^{t-1} + \sum_{p\in P} b_{pi}^{t-1} \quad \forall i \in C_S, t \in T \tag{4}$$

$$\tag{5}$$

where d_i^{t-1} represent the evacuation demand in cell $i \in C_R$ at time $t-1$, and represent evacuee loading and unloading rate. Equation(2)(3)(4) indicate that the vehicles in each cells at time t is equal to the vehicles in previous time step minus the outflow vehicles and plus the inflow vehicles. Evacuation buses load people in source cells with loading rate and drive evacuees to sink cells where buses unload people with same rate. Loading and unloading rate describes the average number of people a bus can pick up in single time step. To simplified the problem and make it consistent with objective function, the loading and unloading rate is divided by average number of people in normal vehicle n. For example, if the loading rate is 6 ppl/Δt and we assume there are 3 people in each vehicle in average. The loading rate becomes to 6 divided by 3, which is 2 veh/Δt.

Flow Restriction Constraints

This set of constraints represents flow propagation constraints that is restricted by the number of vehicles in this cells, the maximum capacity of flow, the remaining capacity of the successor cells. Bus units is captured in traffic flow with ψ_p space occupied by bus p.

$$\sum_{j\in\Gamma^+(i)} y_{ij}^t \le x_i^t \quad \forall i \in C, t \in T \tag{6}$$

$$\sum_{j\in\Gamma^+(i)} y_{ij}^t + \sum_{p\in P} \psi_p L_{pi}^{t+1} \le q_i^t \quad \forall i \in C, t \in T \tag{7}$$

$$\sum_{k\in\Gamma^-(i)} y_{ki}^t + \sum_{p\in P} \psi_p E_{pi}^{t+1} \le q_i^t \quad \forall i \in C, t \in T \tag{8}$$

$$\sum_{k\in\Gamma^-(i)} y_{ki}^t + \sum_{p\in P} \psi_p E_{pi}^{t+1} \le \delta_i^t (N_i - x_i^t - \sum_{p\in P} \psi_p b_{pi}^{t+1}) \quad \forall i \in C, t \in T \tag{9}$$

q_i^t is the flow capacity and N_i is the cell capacity. L_{pi}^t and E_{pi}^t describe the entering and leaving status of bus p at time t. ψ_p is the space a bus p occupies when travel through cells. If bus p enter cell i at time t, there will be ψ_p reduction in the flow capacity. The corresponding reduced capacity is occupied by bus p.

Bus assignment constraints
This constraint makes sure that every bus must be assigned to one of the cells, whether it is idle or working.

$$\sum_{i\in C} b_{pi}^t = 1 \quad \forall p \in P, i \in C, t \in T \tag{10}$$

Entering and leaving cell constraints
These constraints describes the relationship between bus assignment variable b_{pi}^t and entering/leaving variables E_{pi}^t/L_{pi}^t .

$$E_{pi}^t \geq b_{pi}^t - b_{pi}^{t-1} \quad \forall p \in P, i \in C, t \in T \tag{11}$$

$$L_{pi}^t \geq b_{pi}^{t-1} - b_{pi}^t \quad \forall p \in P, i \in C, t \in T \tag{12}$$

E_{pi}^t and L_{pi}^t is determined by the bus assignment variable for consecutive two time steps. If bus p enters cell i at time step t, b_{pi}^t is 0 in previous time step and 1 in the current time step, as a result E_{pi}^t is equal to 1. If bus p leaves cell i at time step t,b_{pi}^t is 1 in previous time step and 0 in the current time step, as a result, L_{pi}^t is equatl to 1. If b_{pi}^t is 0 for both two time steps, it means that bus p is not in cell i. If b_{pi}^t is 1 for both time steps, it means that bus p stays in cell i.

Notice that equation (3) and (4) are in inequality form, which allows $E_{pi}^t = 1$ when bus is not entering and allows $L_{pi}^t = 1$ when bus is not leaving. Such problem is solved when minimizing the objective function, since the value E_{pi}^t and L_{pi}^t generate cost in the objective function.

Capacity constraints
Constraints (13) make sure that the people in the bus is not going to exceed the capacity.

$$0 \leq \sum_{0}^{t} \sum_{i\in C_R} b_{pi}^t - \sum_{i\in C_S} b_{pi}^t \leq C_p \quad \forall p \in P, t \in T \tag{13}$$

Routing logistic constraints
Bus routing constraints make sure that buses travel through network under traffic rule and follow FIFO procedure.

$$\sum_{i\in\Gamma^+(i)} b_{pi}^t \geq b_{pi}^{t-1} - b_{pi}^t \quad \forall p \in P, i \in C, t \in T \tag{14}$$

Constraint(14) make sure that a bus p can only enter successor cells after it leave current cell.

$$b_{pi}^{t+\tau+1} \geq \left(x_i^t - \sum_{0}^{\tau} \sum_{j\in\Gamma(n)} y_{ij}^{t+\tau} \right) / N_i - (1 - E_{pi}^t) M \quad \forall i \in C_I, \quad \tau = 1, 2, 3 ... \tau_i^m, p \in P, t \in T \tag{15}$$

Constraint(15) is First-In-First-Out(FIFO) constraint and only applied for the intermediate cells. This constraints means that after bus p arrives a intermediate cell i, it can only leaves the cell only after all the vehicles that arrives before leaves. That is to say, if a bus p enters a extremely congested cell, it should stay there for a long time, until all the queue before this bus gets out of this cell. τ is a predetermined value which is the maximum time step that a bus is expected to stay in a cell. We can make it as large as possible, but during evacuation, we observed that buses are always given priority, so that a bus will not stay in intermediate cells for more than one time step. We can assume $\tau = 2$ to reduce computational effort, which means the maximum time step a bus could stay in intermediate cell is 2 time step. That does not mean that this constraint is unnecessary. Without FIFO constraints, the model will lose the priority for buses and the buses can go straight through traffic flow without queuing process. When a bus p enter cell i at time t, entering status variable of this bus E^t_{pi} will be 1 at this time step. Start at time step t, until time step $t+\tau$, the bus assignment variable b^t_{pi} of this cell should be 1, until all the vehicles that arrives before bus p leaves. $(1 - E^t_{pi})M$ indicate the equation active once bus p enters a intermediate cell i. From this time step and the following τ time step, the number of vehicles that arrives before bus at time $t + \tau$ is indicated by $x^t_i - \sum\limits_0^{\tau} \sum\limits_{j \in \Gamma(n)} y^{t+\tau}_{ij}$. When the number of rest vehicle before bus is evacuated from this cell, the bus assignment $b^{t+\tau+1}_{pi}$ can be 0 which indicates that bus p can move to the next cell.

4 Illustrative example

We show an illustrative example with a network that consist of 14 cells. In this network, Cell 1 and Cell 2 are source cells. 30 evacuation demand is generated in Cell 1 at the beginning of time horizon. Another 60 evacuation demand generated in Cell 2 at time step 15. Cell 13 and Cell 14 is a sink cell which are safe zone that all evacuees try to get in. Three buses departed from cell 9 to evacuate people from source cells. The evacuation time horizon is 3 minutes, or 30 time steps. The capacity of source cells is infinite. The capacity of intermediate cell is 7 and their flow capacity is 2 vehicles per time step. The example network is presented in figure 1.

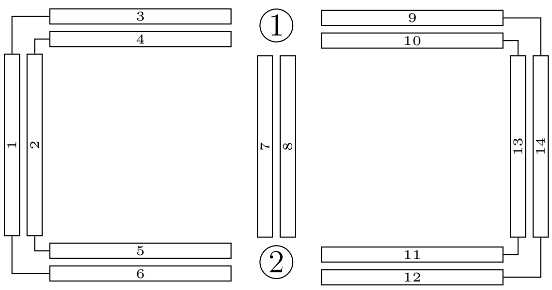

Figure 1: Study Network

The result are shown in Table 1.Bus 1 and 2 depart from cell 9 at time 0. Bus 3 depart one time later because the cell on is congested. Then the buses pick up people at cell 1 and return to cell 14 to unload people. Since in this time period, all evacuees are generated from cell 1. After they drop people in sink cells, Cell 2 start to generates evacuees. Then buses detour to Cell 2 to evacuate people and drop them to cell 13.

$Time$	0	1	2	3	4	5	6	7	8	9	10
$bus0$	9	3	1	1	1	6	12	14	14	14	9
$bus1$	9	3	1	1	1	6	12	14	14	14	9
$bus2$	9	9	3	1	1	1	6	12	14	14	14
$Time$	11	12	13	14	15	16	17	18	19	20	21
$bus0$	7	5	2	2	2	4	10	13	13	13	idle
$bus1$	7	5	2	2	2	4	10	13	13	13	idle
$bus2$	9	7	5	2	2	2	4	10	13	13	13

Table 1: Bus Routing Result

Observation 1 *Buses always obtain priority over other vehicles during the evacuation under the system optimal objective function*

Buses always have priority during evacuation, that means automobile will give way to buses. So that bus will not wait in congested cells. The objective function is to minimize the system travel time experienced by all the vehicles in the network. When buses are evacuating with people on them, the weight of buses is equivalent to the number of evacuees they carry relative to the space it occupies ψ. If there is only 1 person in a bus, the bus will lose priority.

Observation 2 *Buses always fully loaded if possible before they leave pick up location*

Since evacuating with buses is more efficient than with automobiles, they always load people fully before they leave the pick up location. If the people in pick up location is not enough to fill the bus, bus will leave without fully loaded.

Observation 3 *Buses only wait or detour when they encounter congestion that is caused by other buses*

Buses always get priority compared with mobile vehicles. But when they encounter congestion by other buses, they will wait or detour. As seen in time step 1 for bus 3, cell 9 is congested since bus 0 and bus 1 enter this cell at the same time. So bus 3 has to wait for one more time step to enter this cell.

5 Solution Method

The major barrier in applying the Dynamic Bus Routing Problem in real case is computational complexities arising from the high resolution of CTM and integer variables introduced to model bus movements. Computational memory use increases extremely fast with the growing time horizon. The CTM can be solved in a reasonable time, but the integer bus assignment is more difficult.

5.1 Benders decomposition

To reduce computational efforts, we use bender decomposition method to decompose the MILP problem. The master problem(MP)is an integer problem associated with constraints$(10)-(15)$ corresponding to the bus routes assignment problem with pick-up and drop process. The sub problem(SBD) associate with constraints $(1)-(9)$ is dual problem of CTM.

The decomposition algorithm starts with subproblem with initial feasible bus assignments solution $\{b_p^{it}, E_p^{it}, L_p^{it}\}$.
(SP)-D:

$$
\begin{aligned}
max \sum_{t\in T} \Bigg\{ & \sum_{i\in C_R} u_i^{1t} \left(d_i^t - \sum_{p\in P} b_{pi}^{t-1} \right) + \sum_{i\in C_S} u_i^{1t} \sum_{p\in P} b_i^{t-1} \\
& + \sum_{i\in C} u_i^{3t} \left(\sum_{p\in P} L_{pi}^{t+1} - q_i^t \right) + \sum_{i\in C} u_i^{4t} \left(\sum_{p\in P} E_{pi}^{t+1} - q_i^t \right) \\
& + \sum_{i\in C} u_i^{5t} \left(\sum_{p\in P} b_{pi}^t - N_i \right) + \sum_{i\in C_I} \sum_{p\in P} \sum_{\tau=0}^{\tau_{max}} u_{ip}^{6t\tau} \left(-b_{pi}^{t+\tau+1} - M + ME_{pi}^t \right) \Bigg\}
\end{aligned}
$$

$$u_i^{1t} - u_i^{1t+1} + u_i^{2t} - u_i^{5t} \leq 1 \quad \forall i \in C_R, t \in 0...T-1 \tag{16}$$

$$u_i^T + u_i^{2T} - u_i^{5T} \leq 1 \quad \forall i \in C_R \tag{17}$$

$$u_i^{10} - u_i^{11} + u_i^{20} - u_i^{50} \leq 1 \quad \forall i \in C_I \tag{18}$$

$$u_i^{1t} - u_i^{1t+1} + u_i^{2t} - u_i^{5t} + \sum_{p \in P} \sum_{\tau=0}^{\tau_{max}} \frac{1}{N_i} u_{ip}^{6t\tau} \leq 1 \quad \forall i \in C_I, t \in 0...T-1 \tag{19}$$

$$u_i^T + u_i^{2T} - u_i^{5T} \leq 1 \quad \forall i \in C_I \tag{20}$$

$$u_i^{1t} - u_j^{1t+1} + u_i^{2t} - u_i^{5t} \leq 0 \quad \forall i \in C_S, j \in C_S t \in 0...T-1 \tag{21}$$

$$u_i^T + u_i^{2T} - u_i^{5T} \leq 0 \quad \forall i \in C_S \tag{22}$$

$$u_i^{1t} - u_j^{1t} - u_i^{2t} - u_i^{3t} - u_j^{4t} - u_i^{5t} \leq 0 \quad \forall i \in \mathcal{C} \setminus \mathcal{C_I}, t \in T \tag{23}$$

$$\begin{aligned} u_i^{1t} - u_j^{1t} - u_i^{2t} - u_i^{3t} - u_j^{4t} - u_i^{5t} + \frac{1}{N_i} \sum_{c=0}^{\tau_{max}} \sum_{p \in P} u_{pi}^{6(t-c)c} \leq 0 \\ \forall i \in C_I, j \in \Gamma^+(i) t \in T \end{aligned} \tag{24}$$

$$u_i^{2t}, u_i^{3t}, u_i^{4t}, u_i^{5t}, u_{pi}^{6t\tau} \geq 0 \quad \forall i \in C, t \in T \tag{25}$$

According to week duality, we add bender cut to the master problem every iteration until we find the optimal dual variable set u^* and bus assignment solution $\{b^*, E^*, L^*\}$

Application of Bender Decomposition leads to following algorithms.

step 0 . Select ξ for the stop criterion. Iteration time n start with 0. Let upper bound $UB = +\infty$ and lower bound $LB = -\infty$. Develop a initial feasible integer solution $\{b^0, E^0, L^0\}$(we can just assume no buses is applied to initiate).

step 1 Solve the **(SP)-D** and obtain dual variable u^n. Add new bender cut to **(MP)** with constraints(38). Update the UB by the objective value of **(SP)-D**.

(MP)

$$minZ$$

$$
\begin{aligned}
Z \geq \sum_{t\in T} \Bigg\{ & \sum_{i\in C_R} u_i^{1t} \left(d_i^t - \sum_{p\in P} b_{pi}^{t-1} \right) + \sum_{i\in C_S} u_i^{1t} \sum_{p\in P} b_i^{t-1} \\
& + \sum_{i\in C} u_i^{3t} \left(\sum_{p\in P} L_{pi}^{t+1} - q_i^t \right) + \sum_{i\in C} u_i^{4t} \left(\sum_{p\in P} E_{pi}^{t+1} - q_i^t \right) \\
& + \sum_{i\in C} u_i^{5t} \left(\sum_{p\in P} b_{pi}^t - N_i \right) + \sum_{i\in C_I} \sum_{p\in P} \sum_{\tau=0}^{\tau_{max}} u_{ip}^{6t\tau} \, b_{pi}^{t+\tau+1} + M - ME_{pi}^t \\
& + \sum_{0}^{t} \sum_{p\in P} \sum_{i\in C_R} b_{pi}^t - \sum_{i\in C_S} b_{pi}^t \Bigg\}
\end{aligned}
\tag{26}
$$

step 2 Solve the **(MP)** with constraints $(11)-(27)$and bender cut we added from **(SP)-D**. $\{b^n, E^n, L^n\}$ is the new buses assignment solution. Compute the value of **(MP)** Z^n and pdate the LB.

step 3 Substitute $b^{H+1}, E^{H+1}, L^{H+1}$ into dual problem and solve to optimal. Let u^{H+1} be a new optimal solution to dual problem and W^{H+1} be the value of the objective function.

step 4 The iteration stop when $UB = LB$ or the gap between them is less then the criterion ξ. Otherwise, we return to ***step 1*** with new bus assignment value.

An illustrative example is applied by the bender decomposition algorithm. The problem is solved within 1 minutes and used less memory storage. Table 2 is the iteration result by bender decomposition algorithm.

Iteration	Subprob	Masterprob	Iteration	Subprob	Masterprob
1	3292	3814	31	3292	3439
4	3292	3638	32	3292	3432
18	3292	3635	35	3292	3359
21	3292	3635	40	3292	3359
23	3292	3514	42	3292	3323
25	3292	3500	43	3292	3293
26	3292	3479	.	.	.
27	3292	3479	.	.	.
30	3292	3445	105	3292	3292

Table 2: Bender decomposition result

5.2 pareto-optimal cut

Among 105 iteration we can tell from Table 2 that most of them do not help to converge. So we used the Pareto-optimal cuts which is introduced by Magnanti and Wong[22]. For each iteration, after we solve the subproblem, we generate a pareto-optimal cuts that dominate all other cuts. In this way, most of ineffective iterations is eliminated and the model convergence faster.

We used Pareto-optimal cuts on the same problem, the result is presented in table 4.

Iteration	Subprob	Masterprob
1	3292	3732
2	3292	3482
3	3292	3332
4	3292	3292

Table 3: pareto-optimal cut result

The problem only took 4 iteration and less than 1 minutes to solve to optimal. Pareto-optimal cuts makes huge improvement in calculation time and ineffective cuts elimination.

5.3 Dynamic Rolling Horizon Heuristic Method

Even applied Pareto-optimal cut into bender's decomposition to reduce the computational requirement, the real case problem still remain difficult to solve due to the integer master problem. Since the decision variable of bus assignment is cell based and time expended, the real case problem with large network and long time horizon can not be optimally solved even with a very few buses.

This section, we propose a heuristic method based on rolling horizon technique. The structure of our method is built on the heuristic method from He[13], but we make following modification. (1) shortest-path routing is used for buses and we define Shortest Path with Cell Transmission Model(SP-CTM). (2) Each time period is generated dynamically based on a bus assignments. The heuristic method is built on two-stage structure in which, we use a greedy type approach to find the route for buses at the first stage and get the flow pattern of automobiles on the second stage. We summarize key features of the proposed heuristic method in the following section.

Rolling Horizon:

The idea of rolling horizon technique is to use short-time period prediction to make decisions. We can use such predictions to determine bus routing strategy including determining pick up location in the immediate future. The evacuation time horizon get divided as buses start a new trip. Typically for rolling horizon approaches, the time period is equally divided, but in this paper the time period is defined by the start and the end of a trip for every bus. That is, whenever a bus finishes its previously assigned trip, a new time period is created for this bus, and this time period will end at the time when one of the buses' assigned trip comes to an end and needs be assigned to a new trip. The figure 2 show that how our model determines time periods; It starts at the beginning of a trip and ends when this trip is completed.

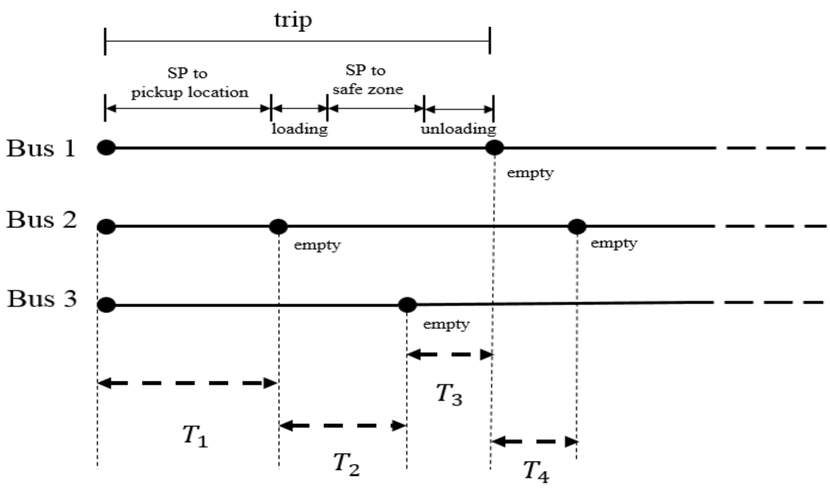

Figure 2: Dynamic Rolling Horizon Time Periods

Bus Assignment:

Bus assignment, or a trip, is comprised of four parts: current location to pick up location; loading evacuees; pick up location to closest shelter location; unloading evacuees. First we solve the LP model of CTM (1) − (9) without any bus assignment. Constraint (2) describe the process that buses picking up people in source cells. So the dual variable of these constraints denotes by u_i^t, informs us how much time step can be reduced one bus is picking up people there for one time step. The summation of the value of u_i^t is denoted by $\beta_i = \sum_{t=a_i}^{b_i} \hat{u}_i^t$, for all $i \in C_R$. This represents the objective function improvement of serving that pick up location. The earliest arrival time a_i is derived from the time of shortest path(SP) from current location to pick up location. The leaving time is the time that bus is fully loaded and is about to leave. This follow the observation 2 that buses always fully loaded.

The algorithm that determines the pick up location for new time period based on dual value of CTM model is presented as follow:

Algorithm 1 PICKUP:D

Input: u_i^t, **Output:** D
for $j \in C_R$ **do**
  $ST_{i,j^{max}} =$ **CTM-SP**(i, j)
  $\beta_j = \sum_{T_q=a_i}^{b_i} \hat{u}_j^{T_q}, \quad \forall j \in C_R$
end for
$\beta_{j^{max}} = Max\{\beta_j | j \in C_R\}$
$D \leftarrow j^{max}$

CTM-Shortest Path:

CTM-SP is used two times in bus assignments: (1) When identifying pick up location, the model creates shortest path to all possible location and find the best one. (2) Sending bus from pick up location to nearest shelter. After identify the pick up location, the buses are assigned to pick up location with SP. The rationale for buses to take SP is from the **observation** 1 that buses always maintain priority. Once a bus is full of people, this bus will drop people to safe area as soon as possible. Current time period is ended when the first bus dropped all people and become empty. With the current assignment of buses, we solve CTM again and start next time period. In next time period, a new pick up location is assign to this bus.

During the process of assignment of buses, buses may be clumped in one cell creating congestion even without automobile. So the flow capacity of the shortest path $\hat{q}_i^t$ is dynamically modified based on previous buses assignments. We formulate Cell Transmission Model based shortest path problem(CTM-SP) to check the feasibility for route and find the shortest path in current stage. The formulation for the model is presented as follow:

CTM-SP

C_O = original cells
C_D = destination cells

$$Maximize \quad \underset{i\in C_D}{} b_i^t \tag{27}$$

$$s.t. \quad \underset{i\in C}{} b_i^t \quad \forall t \in T \tag{28}$$

$$\underset{j\in\Gamma^+(i)}{} b_i^t \geq b_i^{t-1} - b_i^t \quad \forall t \in T, i \in C \tag{29}$$

$$b_i^t - b_i^{t-1} \leq \hat{q}_i^t \quad \forall i \in C \tag{30}$$

$$b_i^{t-1} - b_i^t \leq \hat{q}_i^t \quad \forall i \in C \tag{31}$$

The formulation is developed for one bus as we define a new time period for each bus. The objective of function is to maximize the total time the bus arrive and staying in the destination cells C_D. So that the model will makes bus to get to destination as soon as possible, which is equivalent to the shortest path problem. The first two constraint make sure that the bus travel between cells one at a time. The last two constraints make sure that flow capacity is not violated.

For each bus we have two trips to assign. The first trip is the route from current location to pick up location. In this case, we define the current location as origin O and pick up location as destination D. The second trip is the route from pick up location to safe zone. In this case we define the current location as origin O and all sink cells as destination D. During assignment of buses, the flow capacity $\hat{q}_i^t$ is adjusted based on previous bus assignment.

Framework

The proposed heuristic method is presented below. In this algorithm, the total time horizon T is divided into q time period, denoted by T_q. We note that q is not known prior to finishing and q is determined when the algorithm ends. T_q is determined by the end of a bus trip. Every time a bus become empty, a new time period is created and a new pick up location D are assigned to empty buses. The time for each bus when they become empty is denoted by T_p. The set of empty buses is denoted by P_{empty}.

The algorithm stops when there is no bus needed to be assigned. The following algorithm that generates the set of empty buses P_{empty} at each time period. If we are going to assign bus p in next time period, we add it to P_{empty}. Otherwise, we make it idle and it will never enter P_{empty}.

Algorithm 2 SET: P_{empty}

```
Input:T_p, Output: P_empty
T_min = Min{T_p | ∀p ∈ P}
if T_min = T_total then
   P_empty = ∅
else
   for p ∈ P do
      if T_p = T_min then
         add p into P_empty
      end if
   end for
end if
```

The heuristic method is presented below. For each time period, we pick the best pick up location D for a bus and generate shortest path for it. The number of time for buses picking up people depends on the buses capacity and number of people in source cells. If there is no demand in source cells, we set T_p to T_{total}, and the bus will become idle and will never enter the **SET** function.

Algorithm 3 Heuristic

Initiate $T_p = 0, \forall p \in P \quad \hat{b}_{pi}^t = 0, \forall p \in P, t \in T, i \in N, P_{empty} = \{1, 2, ...P\}$
while $P_{empty} \neq \emptyset$ **do**
  $u_i^t \leftarrow CTM(\hat{b}_{pi}^t)$
  for $p \in P_{empty}$ **do**
    $\beta_D \leftarrow PICKUP(\hat{u}_i^t)$
    if $\beta_D > 0$ **then**
      assign $ST_{i,D}$ to p with time $t_{i,D}$, update $\hat{b}_{pi}^t$
      loading $t_l = Min\{x_D^{T_p}/l, C_p/l\}$, update $\hat{b}_{pi}^t$
      assign $ST_{D,sink}$ to p with time $t_{D,sink}$, update $\hat{b}_{pi}^t$
      unloading $t_u = t_l$, update $\hat{b}_{pi}^t$
      update $T_p = T_p + t_{i,D} + t_l + t_{D,sink} + t_u$
    else
      $T_p = T_{total}$
      bus p become idle
    end if
  end for
  $P_{empty} \leftarrow \text{SET}(T_p)$
end while

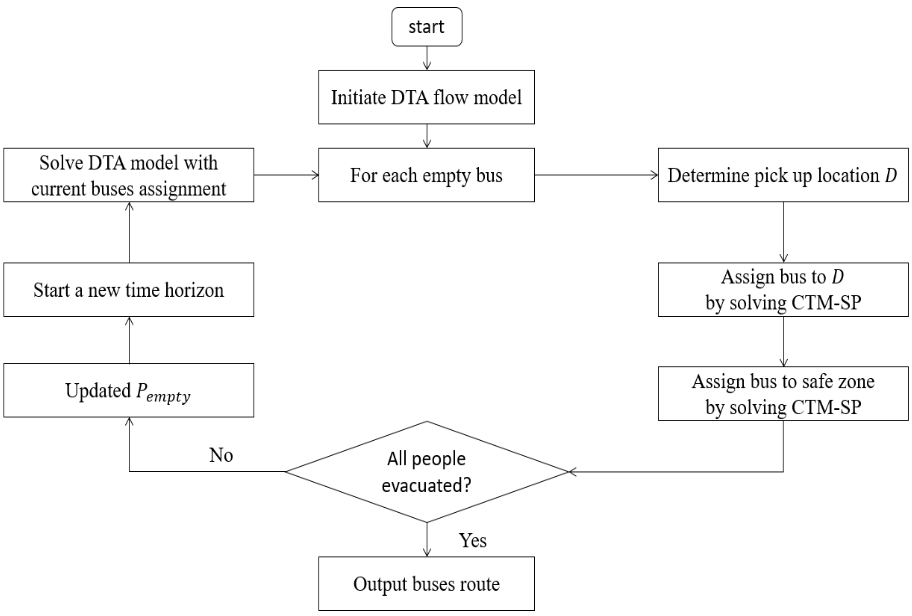

Figure 3: Flowchart of the heuristic method

We apply the algorithm on the small network we used before. The result for heuristic is same as we directly solve it as in table 1, which means for this small case, we find the optimal solution.

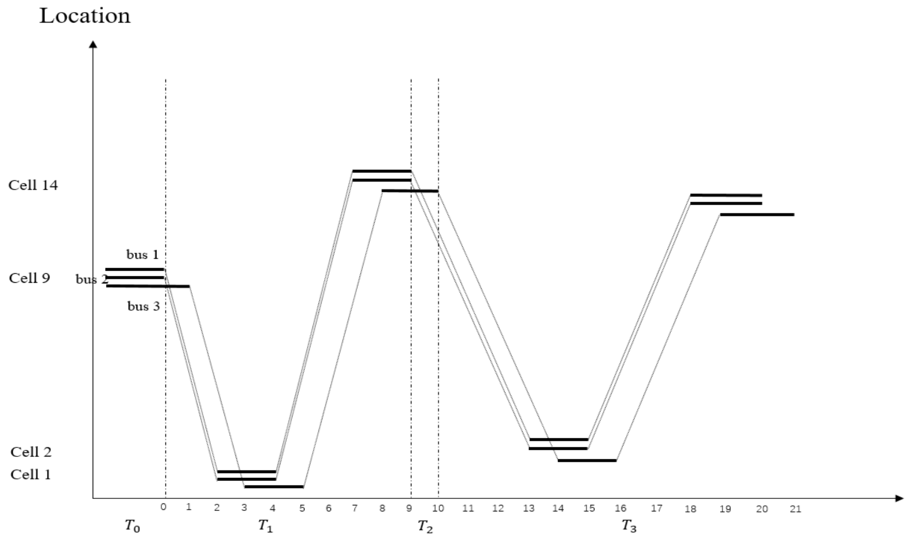

Figure 4: Dynamic Rolling Horizon

The algorithm takes 3 periods total to solve the problem. In the first iteration, bus 1and 2 are assigned to pick up location 1. Due to the congestion from bus 1 and 2, bus 3 departed one step later. The first time period T_1 ended at the time that bus 1 and 2 dropped all the evacuees and the pick up location 2 is assigned in time period T_2. The third time period T_3 generated one step later when bus 3 dropped all evacuees and it is also assigned to cell 2. At the end of this time period, no evacuees are found in source cell. Buses become idle and P_{empty} become empty.

6 Case Study

A evacuation scenario is presented to evaluate the efficiency of heuristic algorithm in this section. The network we used refers to the real case study in He and Peeta[13] which consists of 30 nodes and 107 links and is divided into 342 cells and 504 cell connectors for the application of CTM. Each link has two directions which are modeled with two distinct cells. Traffic in the intersection can go any direction(e.g. turn right, turn left, U turn) with no conflict considered. Cells are sorted into three kinds, cells that represent highways, cells that represent arterials and cells that represent side streets. Evacuees start the evacuation in source cells. All the boundary cells of the network are consider as safe zones which are connected to a "super link". In total, there are 36 source cells and 22 sink cells, the rest of them are intermediate cells.

The cell characteristics are showed in Table 4.

Cell Type	Highway	Arterial	Side Street
Free flow speed(mph)	70	35	15
Time intervel(sec.)	6	6	6
Cell length(feet)	616	308	132
Number of lanes	3	2	1
Maximum link flow	1800	1200	900
Maximum cell flow	9	4	1.5
Maximum number of vehicles per cell	93.8	31.3	6.7

Table 4: pareto-optimal cut result

Evacuation Network

2300 evacuees are generated in 22 source cells, illustrated by ovals in Fig. 4. 10 buses depart from cell 321 to evacuate people from the network. Each has a capacity of 20 and the loading rate is 5 people per time step. We use bus occupancy ratio $\psi_p = 3$ to represent the area a bus occupied when traveling through cells, this means, one bus occupies space equivalent to 3 automobiles. The evacuation planning horizon is 25 minutes, or 250 time steps. Figure 6 presents the corresponding network

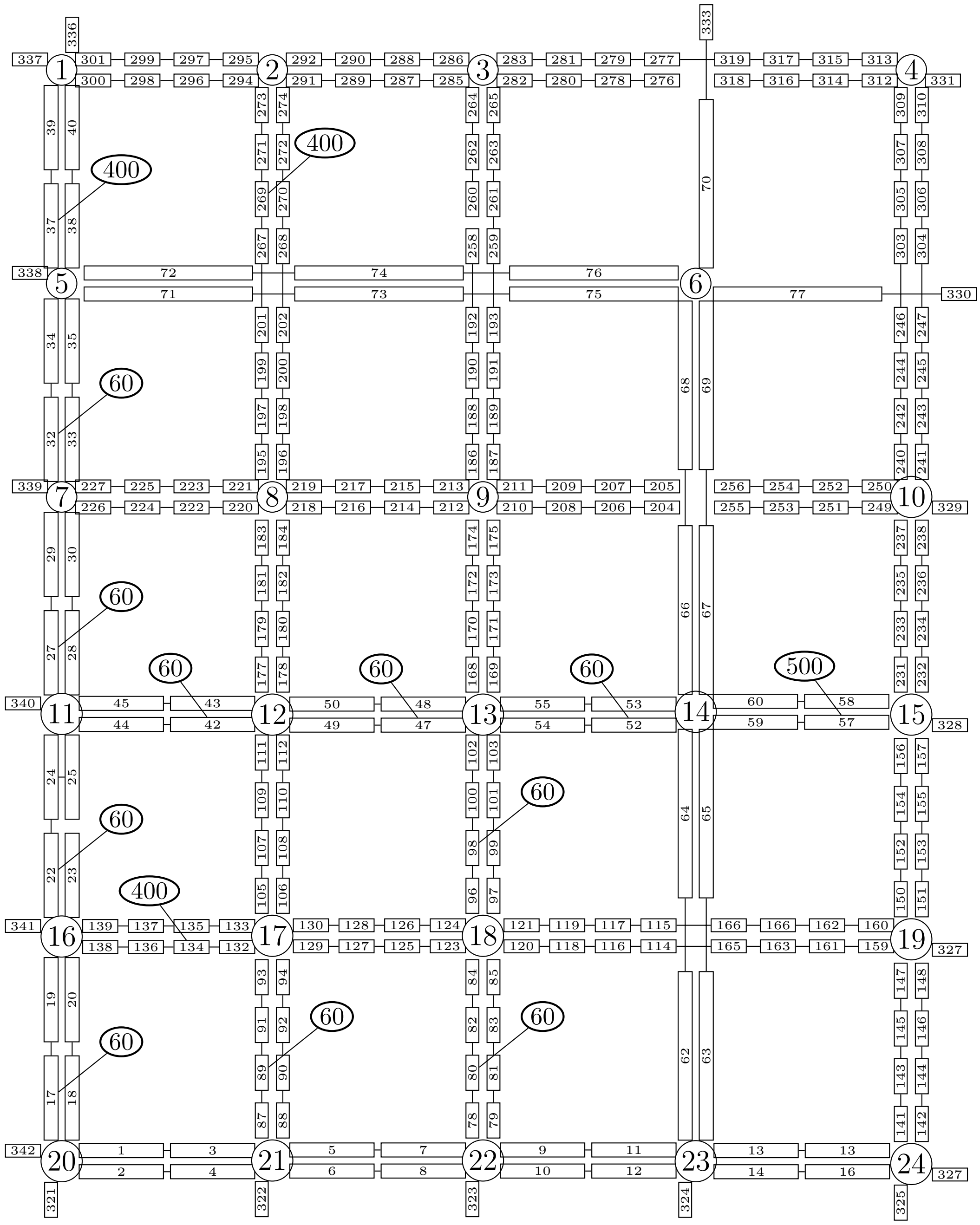

Figure 5: Study Network

The Dynamic Bus Routing Problem is executed on a cluster with 12 cores and 128 GB of memory per node. It takes about 40 minutes and 15 iteration to generate the final result. Since cell 134 is the most congested cell, all bus is assigned to this cell sequentially at the beginning of evacuation. Then buses are distributed through network based on the congestion of each cell. The result is presented in table 5.

iteration	bus0	bus1	bus2	bus3	bus4
1	$321 \to 134$	$321 \to 134$	$321 \to 134$	$321 \to 134$	$321 \to 134$
2	$322 \to 134$	$322 \to 134$			
3			$322 \to 134$	$322 \to 134$	
4					$322 \to 134$
5					
6					
7	$322 \to 57$	$322 \to 57$			
8			$322 \to 134$	$322 \to 134$	
9					$322 \to 57$
10	$328 \to 269$	$328 \to 269$			
11					$328 \to 269$
12			$322 \to 269$	$322 \to 269$	
13					
14					
15					
iteration	bus5	bus6	bus7	bus8	bus9
1	$321 \to 134$	$321 \to 134$	$321 \to 134$	$321 \to 134$	$321 \to 134$
2					
3					
4	$322 \to 134$				
5		$322 \to 57$	$322 \to 57$		
6				$322 \to 57$	$322 \to 57$
7		$328 \to 57$	$328 \to 57$		
8				$328 \to 57$	$328 \to 57$
9	$322 \to 57$				
10		$328 \to 269$	$328 \to 269$	$328 \to 269$	$328 \to 269$
11					
12		$328 \to 269$	$328 \to 269$		
13					
14		$339 \to 269$	$339 \to 269$		
15				$339 \to 269$	$339 \to 269$

Table 5: Heuristic result for case study

The buses assignment result show that, cell 134 is the most congested area. Not just because the large number of neighboring in this cell, but also the large number of neighbor cells that has evacuees. Cell 134 is selected as the pick up location. When there is enough buses to evacuate people from cell 134, buses start to serve cell 57 and cell 269. Since cell 57 has the largest number of evacuees and cell 269 is far away from the safe zone.

Without buses assignment, the system takes 210 time step, which is equal to 21 minutes in real time, to evacuate all the people. We vary the number of buses to see the effect on evacuation time. Following is the table that present the evacuation time, the people evacuated by buses and the computation time.

bus number	Evacuation Time(Minutes)	Percentage people carried by buses(%)	Computation Time(Minutes)
1	19	8%	10.8
2	15.7	12.1%	12.4
3	13.4	16.5%	18.4
4	12.4	22.2%	24.1
5	11.5	26.1%	28.9
6	10.6	28.0%	31.2
7	10.2	30.8%	34.9
8	9.6	36.3%	41.4
9	9.3	35.0%	41.9
10	8.6	38.9%	45.5

Table 6: Evacuation Performance for different number of buses

From table 6 we can tell that, the evacuation time decreased and the number of people increased when number of buses increase. Most of computation time is used to solve the CTM-SP model, since we solve the model for every source cells for each trip.

We further increase the number of buses to see the best evacuation time we can reach. Figure 6 show that with the number of buses increase, the decreasing rate of evacuation time become slower. When the number of buses reach 43, there is no need to assign more buses. The best bus evacuation schedule decrease the evacuation time by 80%.

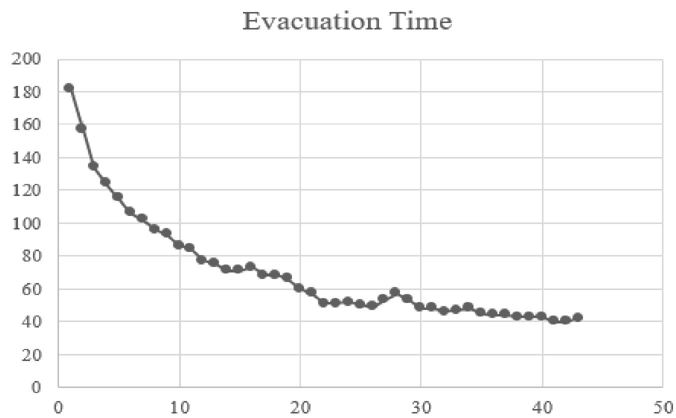

Figure 6: Evacuation time for different number of buses

Figure 7 shows that the number of people carried by buses. Initially the number of people increased when the number of buses increase. After specific number of buses, the number of people in buses tend to be stable and hardly increase.

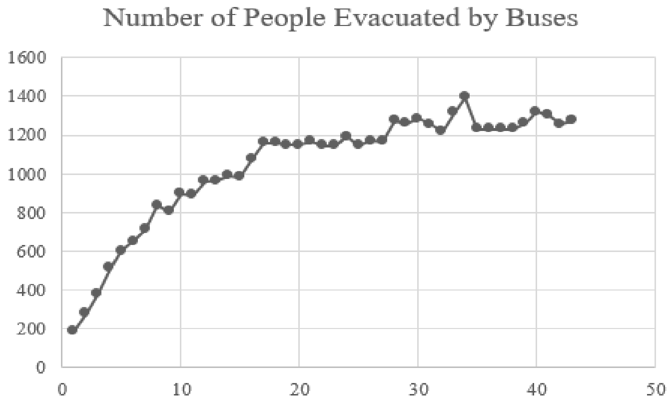

Figure 7: people evacuated by buses for different number of buses

7 Conclusion and future work

In this study, we develop a multiclass dynamic evacuation model where evacuation buses and automobile are scheduled simultaneously. The total travel time is minimized by scheduling the buses to congested areas, while the queuing process, size of buses and loading and unloading process are strictly captured. Adhering to the proposition that bus priority increases the effectiveness of evacuation, buses obtain priority according to the number of people carried. Bender decomposition method cannot solve the real case problem within a reasonable time. To obtain an approximate solution from a real case problem, a two stage dynamic rolling horizon method is developed. The heuristic method aims to determine which areas have the most need and spills evacuation buses over the network. The method is applied on a real-case sized network and prove that buses evacuation with priority can improve evacuation efficiency significantly.

8 Acknowledge

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